(Re)Introducing the K-ratio

Lars Kestner

This draft: March 3rd, 2013

Abstract

I introduced the K-ratio in 1996 as a reward to risk measurement to compliment the popular Sharpe ratio. The K-ratio is calculated by fitting a linear trend series to cumulative returns and estimating the slope and variability of slope. Over the years there have been comments on adjustments factors needed to account for varying number of return observations and return periodicity. In this paper I show that the correct adjustments to the raw K-ratio include dividing by the number of return observations and multiplying by the square root of expected observations in a calendar year.

1. Introduction

The K-ratio is a performance measure that I created in 1996 (Kestner, a and b) that measures the consistency of a strategy's profitability. Like the Sharpe ratio, Sortino ratio, and Calmar ratio[1], the K-ratio is calculated by taking the quotient of reward and risk. Higher K-ratios suggest better performance than lower K-ratios.

2. K-ratio mechanics

The K-ratio calculation begins by creating a time series of cumulative excess returns from a manager or strategy. Each period's returns are summed in an additive or compounded manner. If returns are summed in a geometric manner using compounding, the log of the resulting cumulative return stream must be calculated to transform the series back into linear space.

$$r_t = R_t - R_f$$

$$\text{If additive: } CumRet_1 = 0, \text{ and } CumRet_{t+1} = CumRet_t + r_{t+1}$$

$$\text{If compounded: } CumRet_1 = 1, \text{ and } CumRet_{t+1} = CumRet_t \cdot (1 + r_{t+1})$$

[1] Sharpe ratio as average return by divided by volatility, Sortino ratio as average return divided by semi-volatility, and Calmar ratio as average return divided by maximum drawdown

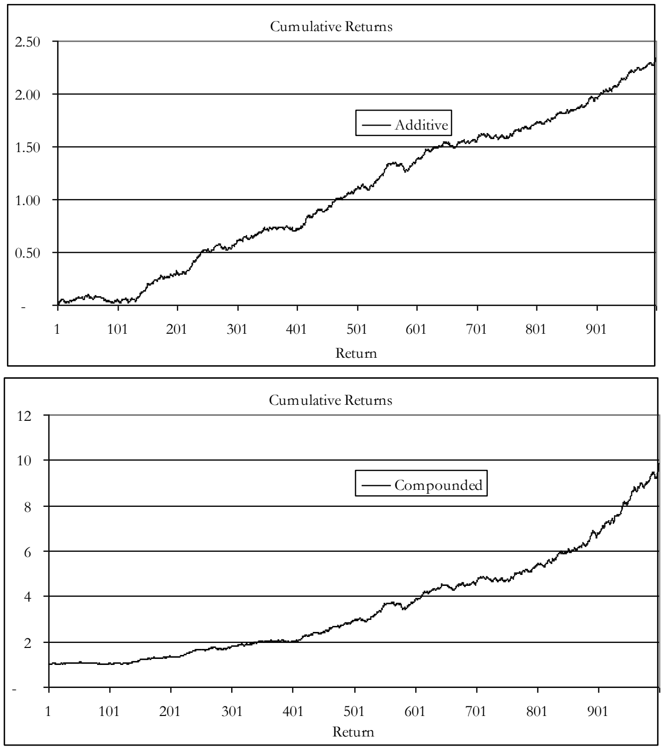

Figure 1. Cumulative returns for additive and compounded processes.

A one factor linear model is created to fit the progress of cumulative returns through time. The only independent variable is a linear trend series beginning at 0 on the first return observation and increasing by one for each additional observation.

$$\text{If additive: } \text{CumRet}_t = b_0 + b_1 \cdot \text{Observation}_t + \varepsilon_t$$

$$\text{If compounded: } \log(\text{CumRet}_t) = b_0 + b_1 \cdot \text{Observation}_t + \varepsilon_t$$

The raw K-ratio is calculated by dividing the b_1 estimate by the standard error of the b_1 estimate. The b_1 estimate and its standard error are determined from ordinary least squares regression techniques. Most would correctly recognize the raw K-ratio as the t-statistic of the b_1 estimate.

$$\text{Raw K-ratio} = \frac{b_1}{SE(b_1)}$$

The reward based numerator of the K-ratio measures the slope of the least squares fitted trendline of cumulative returns. Higher values of b_1 indicate a sharper rise in cumulative returns. Risk of the return stream is calculated using the standard error of the b_1 estimate. Higher standard errors indicate inconsistent performance and lack of stability of the b_1 estimate. Lower standard errors indicate more consistent performance and better stability of the b_1 estimate.

The intuition of the K-ratio is simple. An investor should prefer a manager or strategy that increases wealth (or log(wealth) if returns are compounded) in a straight line manner – with as little deviation as possible. As such, the K-ratio will penalize for both upside and downside volatility, as well as variation in the slope of returns over time.

3. K-ratio adjustment factors

While the raw K-ratio is quite simple to calculate, adjustment factors need to be made in order to compare return streams of differing lengths and varying periodicities. There has been some debate in Kestner (2003) and Becker (2011) over what, if any, scaling factors need to be added to the raw K-ratio calculations. To compare 200 daily returns of Strategy A to 50 weekly returns of Strategy B, we need to adjust the raw K-ratio for both the number of observations as well as the periodicity of those observations. The analysis below will show that the correct adjustments to the raw K-ratio include (1) dividing by the number of return observations and (2) multiplying by the square root of the expected number of return observations in one calendar year.

$$\text{K-ratio} = \frac{b_1}{SE(b_1)} \cdot \frac{\sqrt{per}}{n}$$

where *n* is the number of return observations and *per* corresponds to the expected number of observations in a given calendar year (e.g. 12 for monthly data, 52 for weekly data, and circa 252 for daily data)

To confirm that these scaling factors will lead to comparable K-ratios regardless of the choice of *n* or *per*, I simulate 50 strategies and compare raw K-ratios at different points of both *n* and sampling rates of *per* to study raw K-ratio values. Each of the 50 strategies is generated by pulling returns from a normal distribution with mean return of 0.25% and standard deviation of 1.00%. Each strategy contains 1,000 returns which are summed in an additive manner.

Raw K-ratios are calculated for each strategy ending on successive points from *n*=10 to *n*=1000. That is, raw K-ratios are calculated over the first ten 10 returns including *t*=1, 2, 3, ..., 10, then the first 11 returns including *t*=1, 2, 3,…,11, and so forth until all 1,000 returns from *t*=1, 2, 3,…, 1000 are included. Raw K-ratios are then averaged across all 50 strategies for each ending *n* from 10 to 1,000. That average is plotted in Figure 2 below. As seen, the average raw K-ratio increases in near perfect linear fashion as *n* increases. When we divided the average raw K-ratio by *n*, the relationship diminishes.

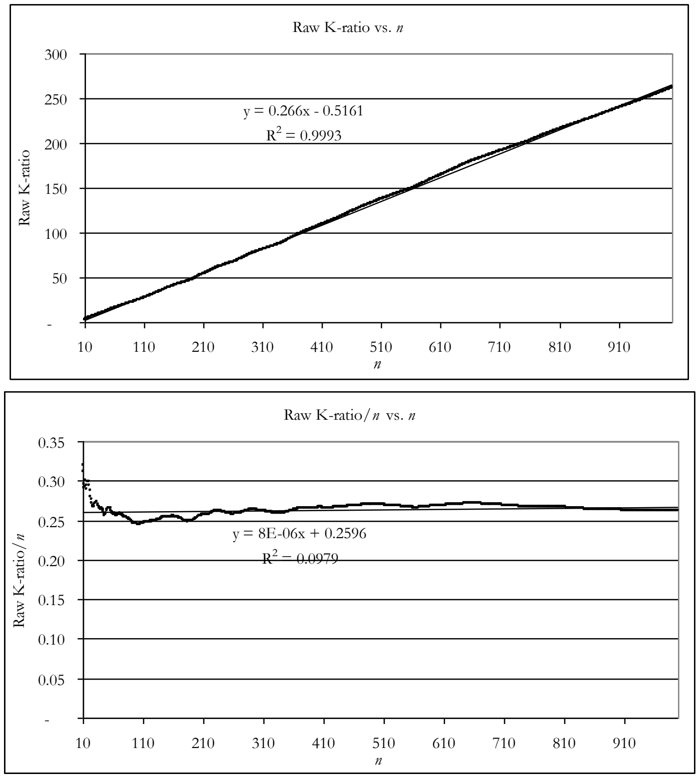

Figure 2. Average raw K-ratios scattered versus *n*; average raw K-ratios divided by *n* then scattered versus *n*.

The second scaling factor results from the varying periodicities of returns that may be sampled. Sharpe ratio calculations are generally scaled to annualized measures. Because average returns scale linearly with sampling period but standard deviation scales proportionately to the square root of the sampling period, a scaling factor is applied to compare Sharpe ratios across return streams with varying periodicities. Sharpe ratios are annualized by multiplying raw ratios by the square root of expected time periods in a year, such that ratios calculated using daily data would be multiplied by $\sqrt{252}$, ratios using weekly data by $\sqrt{52}$, and ratios using monthly data by $\sqrt{12}$.

Similarly, the K-ratio needs to be adjusted depending on the periodicity of sampling of cumulative returns. Figure 3 is a graph of average K-ratios[2] across the 50 simulated strategies with cumulative returns sampled at intervals of (a) 1 period, (b) 3 periods, (c) 9 periods, and (d) 27 periods.

Plotting the average K-ratio versus sampling period, we see a distinct non-linear relationship. Assuming that there are 27 periods in each calendar year, we scale the raw K-ratios by $\sqrt{\frac{27}{\text{samplingperiod}}}$. After making this transformation, we find that the K-ratio is no longer related to the choice of sampling period. As such, when the K-ratio is scaled by multiplying by the square root of expected observations in a calendar year (*per*), the resulting final K-ratio has no bias to the scaling period.

---

[2] The K-ratios for this test have been scaled using the number of returns. That is, K-ratio $= \frac{b_1}{SE(b_1)} \cdot \frac{1}{n}$

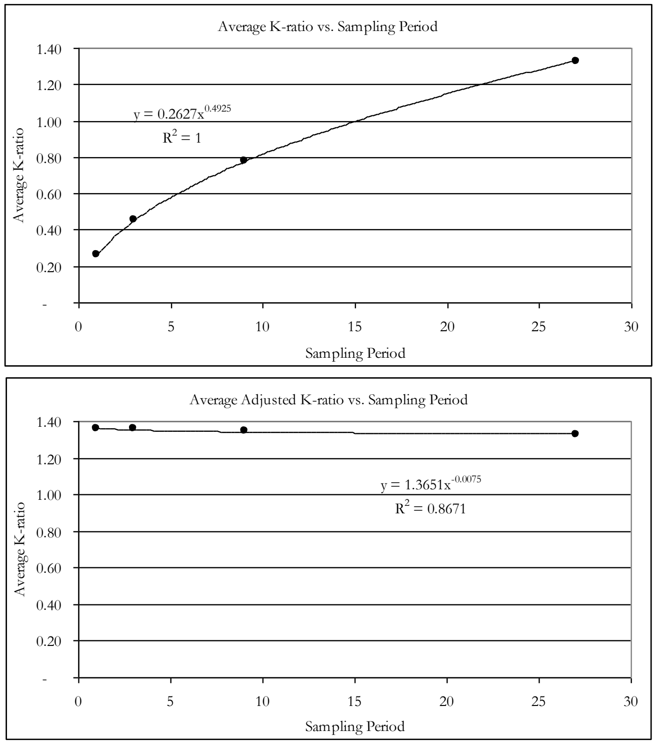

Figure 3. Raw K-ratios plotted versus sampling period length; raw K-ratios multiplied by $\sqrt{\frac{27}{\text{samplingperiod}}}$ then plotted versus sampling period length.

4. Correlated performance measures

Given the ever increasing number of performance measures available to evaluate strategies and managers, a reasonable question to ask is whether one or more ratios are generating highly correlated results. If Performance Measure A and Performance Measure B produce highly correlated results, then the value of using both measures is small. To determine if the K-ratio is generating additional value, we compare K-ratio and Sharpe ratios from the 50 simulated strategies in Figure 4. While the two measures appear to be related, the correlation (R^2 of 31%) suggests that the two measures can vary and that there is additional information content in the K-ratio.

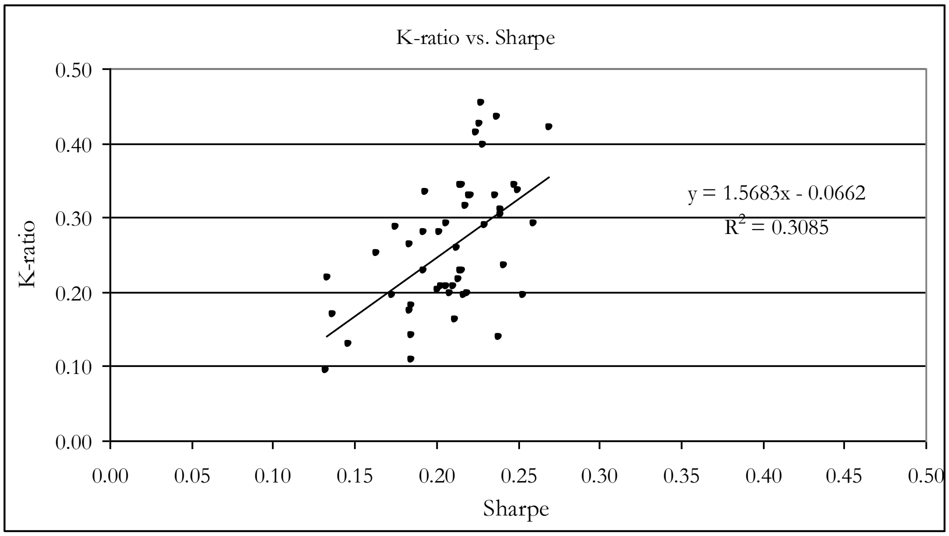

Figure 4. K-ratios plotted versus Sharpe ratios for the 50 simulated strategies.

5. Conclusion

The K-ratio was introduced in 1996 as a reward to risk measurement to compliment the popular Sharpe Ratio. The K-ratio is calculated by fitting a linear trend series to cumulative returns and estimating the slope and variability of slope. Over the years there have been comments on potential adjustments factors needed to account for varying number of return observations and return periodicity. In this paper I show that the correct adjustments to the raw K-ratio include dividing by the number of return observations and multiplying by the square root of expected observations in a calendar year.

References

Becker, Thomas (2011). The Zephyr K-Ratio. Zephyr Associates, Inc. Working paper.

Kestner, Lars N. (1996a). Getting a Handle on True Performance. *Futures*, 25:1, 44-47.

Kestner, Lars N. (1996b). Measuring System Performance. *Stocks and Commodities,* 14:3, 115-117.

Kestner, Lars N. (2003). *Quantitative Trading Strategies*. McGraw Hill.